SECTION 'A'

Q. 1. Answer any *five* of the following :

(a) If M and N are normal subgroups of a group G such that $M \cap N = \{e\}$, show that every element of M commutes with every element of N. 12

(b) Show that $(1 + i)$ is a prime element in the ring R of Gaussian integers. 12

(c) If u, v, w are the roots of the equation in λ and

$$\frac{x}{a+\lambda} + \frac{y}{b+\lambda} + \frac{z}{c+\lambda} = 1, \text{ evaluate } \frac{\partial(x, y, z)}{\partial(u, v, w)}.$$ 12

(d) Evaluate $\iiint \ln(x + y + z)\, dx\, dy\, dz$.

The integral being extended over all positive values of x, y, z such that $x + y + z \leq 1$. 12

(e) If $f(z) = u + iv$ is an analytic function of the complex variable z and $u - v = e^x(\cos y - \sin y)$, determine $f(z)$ in terms of z. 12

(f) Put the following program in standard form :

$$\text{Minimize } z = 25x_1 + 30x_2$$
$$\text{subject to } 4x_1 + 7x_2 \geq 1$$
$$8x_1 + 5x_2 \geq 3$$
$$6x_1 + 9x_2 \geq -2$$

and hence obtain an initial feasible solution. 12

Q. 2. (a) (i) Let H and K be two subgroups of a finite group G such that $|H| > \sqrt{|G|}$ and $|K| > \sqrt{|G|}$. Prove that $H \cap K \neq \{e\}$. 15

(ii) If $f : G \to G'$ is an isomorphism, prove that the order of $a \in G$ is equal to the order of $f(a)$. 15

(b) Prove that any polynomial ring F [x] over a field F is a

30

Q. 3. (a) If f′ and g′ exist for every $x \in [a, b]$ and if $g'(x)$ does not vanish anywhere in (a, b), show that there exists c in (a, b) such that

$$\frac{f(c)-f(a)}{g(b)-g(c)}=\frac{f'(c)}{g'(c)}$$ 30

(b) Show that $\int_0^\infty e^{-t}\, t^{n-1}\, dt$ is an improper integral which converges for $n > 0$. 30

Q. 4. (a) Expand $f(z)=\frac{1}{(z+1)(z+3)}$ in Laurent's series which is valid for

(i) $1 < |z| < 3$

(ii) $|z| > 3$ and

(iii) $|z| < 1$. 30

(b) Use simplex method to solve the following : 30

Maximize $z = 5x_1 + 2x_2$

subject to $6x_1 + x_2 \geq 6$

$4x_1 + 3x_2 \geq 12$

$x_1 + 2x_2 \geq 4$

and $x_1, x_2 \geq 0$.

SECTION 'B'

Q. 5. Answer any *five* of the following :

(a) Formulate partial differential equation for surfaces whose tangent planes form a tetrahedron of constant volume with the coordinate planes. 12

(b) Find the particular integral of

$$x(y-z)\,p + y(z-x)\,q = z(x-y)$$

which represents a surface passing through $x = y = z$. 12

(c) Use appropriate quadrature formulae out of the Trapezoidal and Simpson's rules to numerically integrate $\int_0^1 \frac{dx}{1+x^2}$ with $h = 0.2$

Hence obtain an approximate value of π. Justify the use of a particular quadrature formula. 12

(d) Find the hexadecimal equivalent of $(41819)_{10}$ and decimal equivalent of $(111011.10)_2$.

(e) A rectangular plate swings in a vertical plane about one of its corners. If its period is one second, find the length of its diagonal. 12

(f) Prove that the necessary and sufficient condition for vortex lines and stream lines to be at right angles to each other is that

$$u = \mu \frac{\partial \varphi}{\partial x}, \; v = \mu \frac{\partial \varphi}{\partial y}, \; w = \mu \frac{\partial \varphi}{\partial z}$$

where μ and φ are functions of x, y, z and t. 12

Q. 6. (a) The ends A and B of a rod 20 cm long have the temperature at 30° C and at 80° C until steady state prevails. The temperature of the ends are changed to 40° C and 60° C respectively. Find the temperature distribution in the rod at time t. 30

(b) Obtain the general solution of

$$(D - 3D' - 2)^2 z = 2\, e^{2x} \sin (y + 3x).$$

where $D = \frac{\partial}{\partial x}$ and $D' = \frac{\partial}{\partial y}$ 30

Q. 7. (a) Find the unique polynomial P (x) of degree 2 or less such that P (1) = 1, P (3) = 27, P (4) = 64. Using the Lagrange interpolation formula and the Newton's divided difference formula, evaluate P (1.5). 30

(b) Draw a flow chart and also write a program in BASIC to find one real root of the non linear equation $x = \varphi(x)$ by the fixed point iteration method. Illustrate it to find one real root, correct upto four places of decimals, of $x^3 - 2x - 5 = 0$. 30

Q. 8. (a) A plank of mass M, is initially at rest along a line of greatest slope of a smooth plane inclined at an angle α to the horizon, and a man of mass M′ starting from the upper end walks down the plank so that it does not move. Show that he gets to the other end in time

$$\sqrt{\frac{2M'a}{(M+M')\,g\sin\alpha}}$$

where a is the length of the plank. 30

(b) State the conditions under which Euler's equations of motion can be integrated. Show that

$$-\frac{\partial\varphi}{\partial t}+\frac{1}{2}q^2+V+\int\frac{dp}{\rho}=F(t)$$

where the symbols have their usual meaning. 30
